25. T, since $AB\vec{v} = A\vec{0} = \vec{0}$.

26. T, by Definition 3.2.3.

27. F; Suppose $\vec{v}_2 = 2\vec{v}_1$. Then $T(\vec{v}_2) = 2T(\vec{v}_1) = 2\vec{e}_1$ cannot be $\vec{e}_2$.

28. F; Consider $\vec{u} = \vec{e}_1$, $\vec{v} = 2\vec{e}_1$, and $\vec{w} = \vec{e}_2$.

29. T, since $A^{-1}(AB)A = BA$.

30. T, since both kernels consist of the zero vector alone.

31. T; Consider any basis $\vec{v}_1, \vec{v}_2, \vec{v}_3$ of V. Then $k\vec{v}_1, \vec{v}_2, \vec{v}_3$ is a basis as well, for any nonzero scalar k.

32. F; The identity matrix is similar only to itself.

33. F; Consider $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, but $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$.

34. F; Let $A = I_2$, $B = -I_2$ and $\vec{v} = \vec{e}_1$, for example.

35. F; Let $V = \text{span}(\vec{e}_1)$ and $W = \text{span}(\vec{e}_2)$ in $\mathbb{R}^2$, for example.

36. T; If $A\vec{v} = A\vec{w}$, then $A(\vec{v} - \vec{w}) = \vec{0}$, so that $\vec{v} - \vec{w} = \vec{0}$ and $\vec{v} = \vec{w}$.

37. T; Consider the linear transformation with matrix $[\vec{w}_1 \ \dots \ \vec{w}_n][\vec{v}_1 \ \dots \ \vec{v}_n]^{-1}$.

38. F; Suppose A were similar to B. Then $A^4 = I_2$ were similar to $B^4 = -I_2$, by Example 7 of Section 3.4. But this isn't the case: I_2 is similar only to itself.

39. F; Note that $\mathbb{R}^2$ isn't even a subset of $\mathbb{R}^3$. A vector in $\mathbb{R}^2$, with two components, does not belong to $\mathbb{R}^3$.

40. T; If $B = S^{-1}AS$, then $B + 7I_n = S^{-1}(A + 7I_n)S$.

41. T; Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, for example, with $\ker(A) = \text{im}(A) = \text{span}(\vec{e}_1)$.

42. F; Consider I_n and $2I_n$, for example.

43. T; Matrix $B = S^{-1}AS$ is invertible, being the product of invertible matrices.

44. T; Note that $\text{im}(A)$ is a subspace of $\ker(A)$, so that

$\dim(\text{im } A) = \text{rank}(A) \leq \dim(\ker A) = 10 - \text{rank}(A)$.

45. T; Pick three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ that span V. Then $V = \text{im}[\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$.